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Influence of vertical vibrations on the separation of a binary mixture in a horizontal porous layer heated from below

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ABSTRACT

We study the influence of vertical high-frequency and small-amplitude vibrations on the separation of a binary mixture saturating a shallow horizontal porous layer heated from below. The monocellular flow obtained for a separation ratio $\psi > \psi_{mono} > 0$ leads to a migration of the species towards the two vertical boundaries of the cell. The 2D direct numerical simulations and the linear stability analysis of the averaged governing equations show that the vertical vibrations delay the transition from monocellular flow to multicellular flow. The vibrations also decrease the value of ψ_{mono}, which allows species separation for a wide variety of binary mixtures.

1. Introduction

The problem under consideration concerns the interaction between two phenomena: Soret-driven convection and thermo-vibrational convection in a porous medium.

In binary fluid mixtures subjected to temperature gradients, the thermodiffusion effect induces a mass fraction gradient. In addition to the usual expression for the mass flux $\boldsymbol{J}$ given by the Fick law, an additional part proportional to the temperature gradient is considered so that:

$$\boldsymbol{J} = -\rho D\nabla C - \rho C(1 - C)D_T\nabla T \quad (1)$$

where D is the mass diffusion coefficient, D_T the thermodiffusion coefficient, ρ the density, and C the mass fraction of the denser component.

Thermogravitational diffusion is the combination of two phenomena: convection and thermodiffusion. The coupling of these two phenomena leads to species separation.

In 1938, Clusius and Dickel [1] successfully carried out the separation of gas mixtures in a vertical cavity heated from the side (thermogravitational column, TGC). During the following years, two fundamental theoretical and experimental works on species separation in binary mixtures by thermogravitation were published. Furry et al. [2] (FJO theory) developed the theory of thermodiffusion to interpret the experimental processes of isotope separation. Subsequently, many works appeared, aimed at justifying the assumptions or extending the results of the theory of FJO to the case of binary liquids [3]. Other works were related to the improvement of the experimental devices to increase separation. Lorenz and Emery [4] proposed the introduction of a porous medium into the cavity. Platten et al. [5] used an inclined cavity, keeping the hot plate on the top, to increase separation.

Elhajjar et al. [6] used a horizontal cavity heated from above with temperature gradients imposed on the horizontal walls to improve the separation process with the use of two control parameters.

Double-diffusive convection caused by temperature and concentration gradients in a porous medium has been widely studied due to its numerous fundamental and industrial applications. Some examples of interest are the migration of moisture in fibrous insulation, the transport of contaminants in saturated soil, drying processes or solute transfer in the mushy layer during the solidification of binary alloys. A review of the recent works in this field is given by Nield and Bejan [7]. Soret-driven convective effects cannot be neglected in many industrial processes. Sovran et al. [8] studied the onset of Soret-driven convection in an infinite horizontal porous layer. For a cell heated from below and for a positive separation ratio $\psi > 0$ they showed that the first primary bifurcation is a stationary one. Using a regular perturbation method, in the case of long wave disturbances (i.e. $k = 0$), they found the set of critical parameters $Ra_c = 12/(Le\psi)$, $k_c = 0$ for $\psi \geqslant \psi_{mono} = 1/[(40/51)Le - 1]$.

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Nomenclature

A	aspect ratio of the cavity
a	thermal diffusivity of the mixture $a = \lambda^*/(\rho c)_f$
b	amplitude of vibrations (m)
C	mass fraction of the denser component of the mixture
C_i	initial mass fraction of the denser component of the mixture
D^*	mass diffusion coefficient (m^2/s)
Da	Darcy number $Da = K/H^2$
D_T^*	thermodiffusion coefficient (m^2/s K)
H	height of the cavity (m)
k	wave number
k_c	critical wave number for the bifurcation from the equilibrium solution
k_{c2}	Critical wave number associated with transition from monocellular to multicellular flow
K	permeability of the porous medium (m^2)
L	width of cavity (m)
Le	Lewis number $Le = a/D^*$
P	pressure of fluid (Pa)
Pr	Prandtl number $Pr = (\upsilon(\rho c)_f)/\lambda^*$
Ra	thermal Rayleigh number $Ra = [KHg\beta_T\Delta T(\rho c)_f]/(\lambda^* \nu)$
Ra_c	critical Rayleigh number associated with transition from equilibrium solution to monocellular flow
Ra_{c2}	critical Rayleigh number associated with transition from monocellular to multicellular flow
R	$R = (b\varpi^2)/g$
R_v	$R_v = (Ra^2R^2B)/(2(B^2\omega^2 + 1))$
T	temperature (K)
S	separation
t	nondimensional time
t'	dimensional time (s)
$\mathbf{V}$	velocity of the flow (m/s)
u and v	velocity components (m/s)

Greek symbols

ψ	separation ratio
ε	normalized porosity
ρ	density of the mixture (kg/m^3)
ϖ	dimensional frequency
β_T	thermal expansion coefficient (1/K)
β_C	mass expansion coefficient
σ	temporal amplification of perturbation
ψ_{mono}	separation ratio beyond which flow at onset of convection is monocellular
ω	nondimensional frequency of vibrations
λ^*	effective thermal conductivity of the porous medium-mixture system (W/m K)
$(\rho c)_f$	volumetric heat capacity of the mixture (J/m^3 K)
$(\rho c)^*$	volumetric heat capacity of porous medium-mixture system (J/m^3 K)
ε^*	porosity of porous medium
ν	kinematic viscosity of mixture (m^2/s)
ω_{c2}	critical frequency associated with transition from monocellular to multicellular flow

Many works have been devoted to thermo-vibrational convection in porous media. Khallouf et al. [9] considered a square differentially heated cavity filled with a porous medium saturated by a pure fluid and subjected to linear harmonic oscillations in the vertical direction. In their study, the authors used a Darcy–Boussinesq model and a direct formulation. In the case of a horizontal porous layer saturated by a pure fluid, heated from below or from above, Zen'kovskaya and Rogovenko [10], Bardan and Mojtabi [11], used the Darcy model including the non-stationary term and adopted the time-averaged equations formulation to study the influence of high-frequency, small-amplitude vibrations on the onset of convection. They found that vertical vibrations stabilize the rest solution. Charrier Mojtabi et al. [12] investigated the influence of vibrations on Soret-driven convection in a horizontal porous cell heated from below or from above. They showed that the vertical vibrations had a stabilizing effect while the horizontal vibrations had a destabilizing effect. Thermo-vibrational convection in a fluid medium has received more attention than thermo-vibrational convection in a porous medium (Gershuni et al. [13], Gershuni and Lyubimov [14], etc.). It is well known that high-frequency accelerations induced by crew activities in microgravity platforms (g-jitter), can produce drastic disturbances during the experiments in space as, for instance, in solidification processes during which mushy zones, modeled as porous media, are produced. The g-jitter, which can be represented by a unidirectional harmonically oscillating small-amplitude acceleration field (Alexander [15]), leads to a non-zero mean flow which may have an important effect on the average heat transfer. High-frequency vibrations can also significantly alter earth-bound experiments. In the case of weightlessness, only the specific thermo-vibrational mechanism is responsible for instabilities. Under a gravity field, both thermo-vibrational and thermo-gravitational mechanisms occur. All these previous works show that it is important to study the control of convective motions by vibration effects either (i) in a single constituent or a binary fluid or (ii) in a porous medium saturated by a single constituent or a binary fluid. Gershuni et al. [16,17], analyzed the stability of mechanical quasi-equilibrium or mechanical equilibrium of a binary mixture horizontal layer subjected to a vertical temperature gradient, under a high-frequency vibrational field, when Soret-effect is taken into account. Recently, Shevtsova et al. [18] produced a benchmark of numerical solutions of the vibrational convection problem with Soret effect in a cubic rigid cell filled with water (90%) and isopropanol and subjected to a temperature gradient between two opposite lateral walls. In the present paper, we use the same formulation as the one used by Charrier Mojtabi et al. [12] for a shallow porous cavity saturated by a binary mixture and heated from below. We verify that it is possible to carry out the species separation of a binary mixture in this geometrical configuration, and that the vibrations can be used to delay the loss of stability of the monocellular flow, which allows separation at a higher Rayleigh number. We consider the case of high-frequency, small-amplitude vibrations, so that a formulation using time-averaged equations can be used. The results of the linear stability analysis of the mechanical equilibrium and the monocellular flow in an infinite porous layer heated from below, in the case of a separation ratio $\psi > \psi_{mono} > 0$, are corroborated by the direct numerical simulations.

2. Mathematical formulation

We consider a rectangular cavity with aspect ratio $A = L/H$, where H is the height of the cavity along the vertical axis and L is the width along the horizontal axis. The aspect ratio is assumed infinite in the stability analysis. The cavity is filled with a porous medium saturated by a binary fluid for which the Soret effect is taken into account. The impermeable horizontal walls are kept at different, uniform temperatures: T_1 for $z = 0$ and T_2 for $z = H$, with $T_1 > T_2$. The vertical walls ($x = 0$, $x = L$) are impermeable and adiabatic. All the boundaries are assumed rigid. The cavity is subjected to linear harmonic oscillations in the vertical direction (amplitude b and dimensional frequency ϖ). For the governing equations, we adopt the Boussinesq approximation and Darcy equation for which the non-stationary term is taken into account.

We set all the properties of the binary fluid constant except the density ρ in the buoyancy term, which depends linearly on the local temperature and mass fraction:

$$\rho = \rho_r[1 - \beta_T(T - T_r) - \beta_C(C - C_r)] \tag{2}$$

where ρ_r is the fluid mixture density at temperature T_r and mass fraction C_r. β_T and β_C are the thermal and concentration expansion coefficients, respectively.

When we consider the referential related to the oscillating system, the gravitational field $\mathbf{g}$ is replaced by $\mathbf{g} + b\varpi^2\sin(\varpi t')\mathbf{e_z}$ where $\mathbf{e_z}$ is the unit vector along the vertical axis (vibration axis) and t' the dimensional time.

Thus the dimensionless governing conservation equations for mass, momentum, energy and chemical species, where the Soret effect is taken into account are:

$$\begin{aligned}
&\nabla.\mathbf{V} = 0\\
&B\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} = -\nabla P + Ra(T + \psi C)(1 - R\sin(\omega t))\mathbf{e_z}\\
&\frac{\partial T}{\partial t} + \mathbf{V}.\nabla T = \nabla^2 T\\
&\varepsilon\frac{\partial C}{\partial t} + \mathbf{V}.\nabla C = \frac{1}{Le}\left(\nabla^2 C - \nabla^2 T\right)
\end{aligned} \tag{3}$$

where $B = Da(\rho c)_f/[(\rho c)^*\varepsilon Pr]$ is the inverse of the Vadasz number ($B = 1/Va$)and $R = b\varpi^2/g$. $Da = K/H^2$ is the Darcy number and K the permeability of the porous medium. The reference scales are H for the length, $H^2/(\lambda^*/(\rho c)^*)$ for the time (where λ^* and $(\rho c)^*$ are, respectively, the effective thermal conductivity and volumetric heat capacity of the porous medium), a/H for the velocity with $a = \lambda^*/(\rho\ c)_f$ (a is the effective thermal diffusivity), $\Delta T = T_1 - T_2$ for the temperature and $\Delta C = -\Delta T C_i(1 - C_i)D_T^*/D^*$ for the mass fraction where C_i, D_T^*, D^* are the initial mass fraction, the thermodiffusion and the mass diffusion coefficients of the denser component, respectively. The dimensionless temperature and mass fraction are respectively defined by : $T = (T^* - T_2)/\Delta T$, $C = (C^* - C_i)/\Delta C$.

The dimensionless boundary conditions are:

$$\begin{aligned}
&T = 1 \text{ for } z = 0; \quad T = 0 \text{ for } z = 1; \quad \tfrac{\partial T}{\partial x} = \tfrac{\partial C}{\partial x} = 0 \text{ for } x = 0, A;\\
&\nabla C\cdot\mathbf{n} = \nabla T\cdot\mathbf{n} \text{ for } z = 0, 1; \mathbf{V}\cdot\mathbf{n} = 0\ \forall M \in \partial\Omega
\end{aligned} \tag{4}$$

The problem under consideration depends on eight non-dimensional parameters: the thermal Rayleigh number, $Ra = K\ g\beta_T H\Delta T\ (\rho c)_f/(\lambda^*\upsilon)$, $R = b\varpi^2/g$, the separation ratio $\psi = -(\beta_T/\beta_c)(D_T^*/D^*)\ C_i(1 - C_i)$, the Lewis number $Le = a/D^*$, the normalized porosity $\varepsilon = \varepsilon^*(\rho c)_f/(\rho c)^*$ (where ε^* is the porosity), the dimensionless frequency ω, the aspect ratio A and the factor B.

In the momentum equation the term $B\partial V/\partial t$ is usually neglected since B is of order 10^{-6}. However, in our problem, high-frequency vibrations cause very large accelerations, making it necessary to consider this non-stationary term [10].

3. The averaged equations

In the limiting case of high-frequency and small-amplitude vibrations, the averaging method can be applied to study thermal vibrational convection [14]. According to this method, each field ($\mathbf{V}$, P, T, C) is subdivided into two parts: the first part varies slowly with time (i.e. the characteristic time is large with respect to the period of the vibrations) and the second one varies quickly with time (i.e. the characteristic time is of the order of magnitude of the vibrational period):

$$\begin{aligned}
&\mathbf{V} = \mathbf{V}^*(t) + \mathbf{u}'(\omega t); P = P^*(t) + p'(\omega t); T = T^*(t) + \theta'(\omega t);\\
&C = C^*(t) + c'(\omega t)
\end{aligned}$$

Here, $\mathbf{V}^*$,P^*,T^*,C^* are the averaged fields (i.e. the mean value of the field calculated over the period $\tau = 2\pi/\omega$) of the velocity, pressure, temperature and mass fraction. The decoupling between the pulsational parts of the velocity and the pressure is obtained by using a Helmhotz decomposition of $(T^* + \psi C^*)\mathbf{e_z}$.

$(T^* + \psi\mathbf{C}^*)\mathbf{e_z} = \mathbf{W} + \nabla\xi$ where $\mathbf{W}$ is the solenoidal part of $(T^* + \psi\mathbf{C}^*)\mathbf{e_z}$ satisfying $\nabla\cdot\mathbf{W} = 0$.

Thus the averaged flow equations are:

$$\begin{aligned}
&\nabla.\mathbf{V}^* = 0\\
&B\frac{\partial \mathbf{V}^*}{\partial t} + \mathbf{V}^* = -\nabla P^* + Ra(T^* + \Psi C^*)\mathbf{e_z} + Rv\Big[\mathbf{W}\cdot\Big(\nabla T^* + \frac{\Psi}{\varepsilon^*}\nabla C^*\Big)\Big]\mathbf{e_z}\\
&\frac{\partial T^*}{\partial t} + \mathbf{V}^*\cdot\nabla T^* = \nabla^2 T^*\\
&\varepsilon\frac{\partial C^*}{\partial t} + \mathbf{V}^*\cdot\nabla C^* = \frac{1}{Le}\left(\nabla^2 C^* - \nabla^2 T^*\right)\\
&(T^* + \psi\mathbf{C}^*)\mathbf{e_z} = \mathbf{W} + \nabla\xi; \quad \nabla\cdot\mathbf{W} = 0
\end{aligned} \tag{6}$$

In addition to the boundary conditions (4) applied to the mean fields, we consider: $\mathbf{W}\cdot\mathbf{n} = 0$ on $\partial\Omega$.

The modified vibrational Rayleigh number $Rv = (Ra^2R^2B)/(2(B^2\ \ \omega^2 + 1))$ characterizes the intensity of the vibrations.

4. Linear stability of the equilibrium solution in an infinite horizontal porous layer

The stability of the equilibrium solution was studied by Charrier-Mojtabi et al. [12]. They restricted their study to the case $Le = 2$ for which the fluid considered is in the gaseous state, and so the Dufour effect should be taken into account. We extended this study to the case of a high Lewis number and we focused on the transition from the equilibrium solution to the monocellular flow obtained for binary mixtures with positive separation ratio: $\psi > \psi_{mono} > 0$.

This problem admits a mechanical equilibrium solution characterized by:

$$\mathbf{V_0^*} = \mathbf{0}; \quad T_0^* = 1 - z; \quad C_0^* = cst - z; \quad \mathbf{W} = \mathbf{0} \tag{7}$$

In order to analyze the stability of this conductive solution, we introduced the perturbation of the vertical velocity component w, the perturbation of the vertical component of $\mathbf{W}$, w_2, and the perturbations of temperature,θ, and concentration, c. We assumed that the perturbations (w, w_2, θ, c) were small.

We introduced a new function $\eta = c - \theta$, in order to more easily take into account the boundary conditions on θ and c at $z = 0$ and 1.

The linear stability equations were solved using the 4th order Galerkin method.

For $\psi > 0$ the first bifurcation is stationary. The factor B was set to 10^{-6} and $\varepsilon = 0.5$. For $Le = 100$ we determined the bifurcation diagrams, $Ra_c = f(\psi)$ and $k_c = f(\psi)$, where Ra_c and k_c are respectively the critical thermal Rayleigh number and the critical wave number in the infinite horizontal direction. The results are illustrated in Figs. 1 and 2, for the case $Le = 100$, $\varepsilon = 0.5$ and for $Rv = 0,10,50$. For a layer heated from below, it can be noted that Ra_c increases with Rv whereas k_{cs} decreases with Rv. For $\psi > 0$, when Rv increases, the value ψ_{mono} of the separation ratio beyond which the critical wave number vanishes (i.e. $k_c = 0$), decreases. Table 1 shows the influence of vibrations (Rv) on ψ_{mono}. We note that ψ_{mono} decreases and becomes close to 0 when Rv increases. For $Le = 100$, we obtain $\psi_{mono} = 0.0129$ without vibration and $\psi_{mono} = 0.0033$ for $Rv = 100$. This value of ψ is very small and most binary liquids have a separation ratio higher than this value. So, by adding vibrations, we can use the horizontal cell to separate most binary mixtures. Using a regular perturbation method in the case of long wave disturbances (i.e.$k = 0$), we showed that, for $\psi > \psi_{mono}$, the first primary bifurcation is a stationary one and the critical thermal Rayleigh number is: $Ra_c = 12/(Le\psi)\ \forall Rv$. Furthermore, we noticed

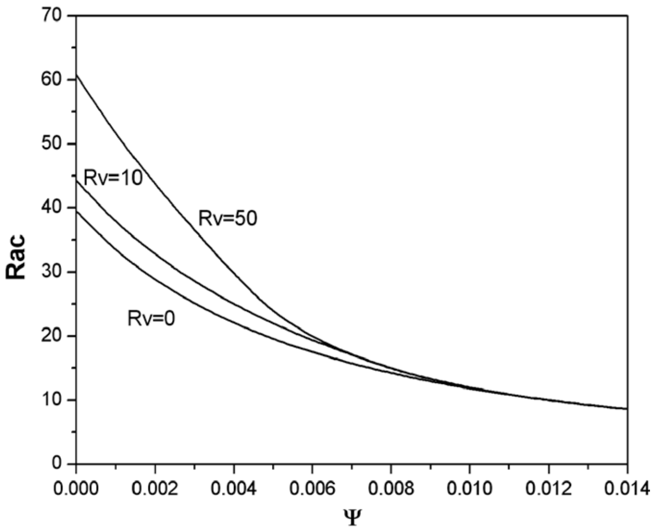

Fig. 1. Critical Rayleigh number at the onset of convection versus separation ratio for $Le = 100$, $B = 10^{-6}$, and $\varepsilon = 0.5$.

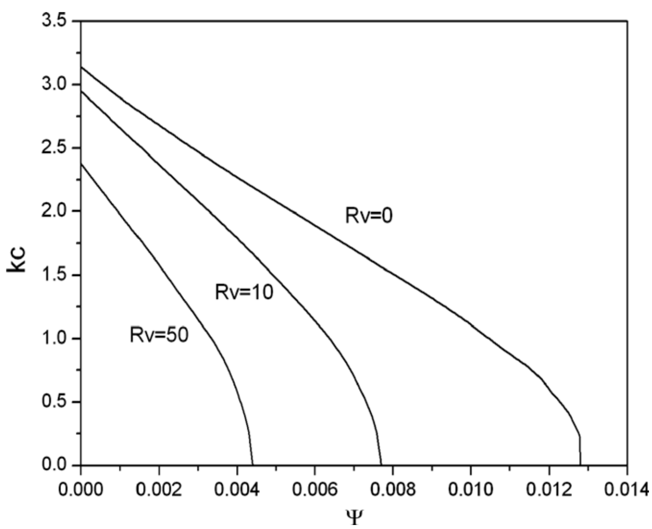

Fig. 2. Critical wave number at the onset of convection versus separation ratio for $Le = 100$, $B = 10^{-6}$, and $\varepsilon = 0.5$.

Table 1
Effect of vibrations on the value of the separation ratio ψ_{mono} beyond which the flow at the onset of convection becomes monocellular for $\varepsilon = 0.5$, $B = 10^{-6}$

Rv	ψ_{mono} for $Le = 100$	ψ_{mono} for $Le = 30$
0	0.0129	0.0444
10	0.0077	0.0257
20	0.0062	0.0205
30	0.0054	0.0177
40	0.0048	0.0159
50	0.0044	0.0146
60	0.0041	0.0136
70	0.0039	0.0128
80	0.0037	0.0121
90	0.0035	0.0115
100	0.0033	0.0110

that when the vibrational Rayleigh number, Rv, increased, the value of the separation ratio ψ_{mono} beyond which the wave number $k_c = 0$ decreased. To conclude, vertical vibrations have a stabilizing effect on the convective motions whereas they do not modify the value of Ra_c for the flow associated with the long wave mode (i.e. $k_c = 0$). The same results can be observed in a horizontal binary fluid layer [16].

5. Analytical solution of the monocellular flow

In the case of a shallow cavity $A \gg 1$, we considered the parallel flow approximation used by Cormack et al. [19]. The basic flow, denoted with a subscript 0, is then given as follows:

$$\mathbf{V}_0 = U_0(z)\mathbf{e_x};\, T_0 = bx + f(z);\, C_0 = mx + g(z);\, \mathbf{W}_0 = W_{10}(z)\mathbf{e_x} \qquad (8)$$

For the stationary state, when the above mentioned assumptions are made and the corresponding boundary conditions are considered, we obtain the velocity, temperature and concentration fields:

$$\begin{cases} T_0 &= 1 - z \\ U_0 &= Ram\ \Psi(1/2 - z) \\ C_0 &= mx + \left(m^2 RaLe\Psi\left(3z^2 - 2z^3\right)\right)/12 - z \\ & \quad -\left(m^2 RaLe\Psi\right)/24 + (1 - mA)/2 \\ m &= \pm\sqrt{(10LeRa\Psi - 120)}/(LeRa\Psi) \\ W_{10} &= (b + \psi m)(1/2 - z) \end{cases} \qquad (9)$$

The expressions obtained for the velocity, the temperature and the concentration fields are similar to those obtained in [20]. This means that the basic state, corresponding to the monocellular flow, is independent of the high-frequency and small-amplitude vibrations.

The separation, $S = mA$, is defined as the difference in mass fraction of the denser component in the vicinity of the left and the right vertical walls of the cell. Then the maximum separation is obtained for $Ra = 24/(Le\Psi)$. This value is denoted Ra_{opt}.

Fig. 3 presents the separation value versus $RaLe\psi$ obtained analytically and numerically for an arbitrary moderate chosen value of Rv that does not destabilize the monocellular flow. We verified numerically, solving the averaged Eqs. (6) for $Rv = 50$, that the monocellular flow remains stable and leads to the exact separation value obtained analytically with Eq. (9). It can be seen that the separation has a maximum corresponding to the optimal coupling between thermodiffusion and convection. For small values of the Rayleigh number, the convection is weak and the separation is mainly caused by thermodiffusion. For high values of the Rayleigh number, the convection is strong and the flow increases the mixing

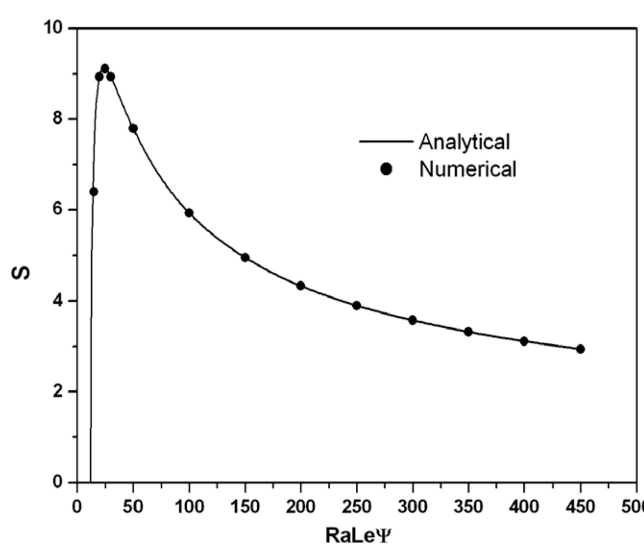

Fig. 3. Separation versus $RaLe\psi$ for $Rv = 50$. $(RaLe\psi)_{\text{opt}} = 24$.

of the species leading to a small separation. It can be noted that, due to the reference scale used for the concentration field, the separation S may be higher than 1.

6. Linear stability analysis of the monocellular flow

We studied the stability of the monocellular solution in order to confirm that the species separation could occur in a horizontal porous cell heated from below. For this study, we write the governing equations using the perturbations of the velocity $\mathbf{v}$, temperature θ, pressure p, mass fraction c, and solenoidal field $\mathbf{w}$:

$$\mathbf{v} = \mathbf{V} - \mathbf{V_0}, \quad \theta = T - T_0, \quad c = C - C_0, \quad p = P - P_0,$$
$$\mathbf{w} = \mathbf{W} - \mathbf{W_0}.$$

The second-order terms are neglected; we obtain the linear equations where the unknown functions are the perturbations.

To take the boundary conditions for temperature and concentration at $z = 0$ and 1 into account more easily, we introduce the new variable $\eta = c - \theta$.

The disturbances are developed in normal modes $(w,\theta,\eta,w_2) = (w(z), \theta(z),\eta(z),w_2(z))e^{(Ikx+\sigma t)}$ and we obtain the following equation system:

$$\begin{cases} (B\sigma+1)\left(D^2-k^2\right)w + Rak^2(\theta(1+\Psi)+\Psi\eta) + R_v[kIDw_2m(\psi/\varepsilon) \\ +w_2k^2(DT_0+(\psi/\varepsilon)DC_0) + W_{10}Ik^3((1+\psi/\varepsilon)\theta+(\psi/\varepsilon)\eta)\Big] = 0 \\ \left(D^2-k^2\right)\theta - \sigma\theta + w - IkU_0\theta = 0 \\ (1/Le)Ik\left(D^2-k^2\right)\eta - \varepsilon\sigma Ik(\eta+\theta) + k^2U_0(\eta+\theta) + mDw - IkwDC_0 = 0 \\ \left(D^2-k^2\right)w_2 + k^2((1+\psi)\theta+\psi\eta) = 0 \end{cases} \quad (10)$$

where $D = \partial/\partial z$, k is the horizontal wave number, $\sigma = \sigma_r + I\sigma_i$ is the temporal amplification of the perturbation, $I^2 = -1$, w is the vertical component of the velocity and w_2 the vertical component of $\mathbf{w}$.

The corresponding boundary conditions are:

$$w = 0, \theta = 0, \frac{\partial\eta}{\partial z} = 0, w_2 = 0 \quad \text{at} \quad z = 0, 1 \quad (11)$$

The resulting linear problem is solved using the 4th order Galerkin method. The perturbation quantities are chosen as follows:

$$w(z) = \sum_{i=1}^{N} a_i \sin(i\pi z); \quad \theta(z) = \sum_{i=1}^{N} b_i \sin(i\pi z);$$
$$\eta(z) = \sum_{i=0}^{N-1} c_i \cos(i\pi z); \quad w_2(z) = \sum_{i=1}^{N} d_i \sin(i\pi z)$$

The critical values of the Rayleigh number and the wave number were obtained for a stationary and a non-stationary bifurcation. For the values of ψ and Le studied, the critical Rayleigh number leading to stationary bifurcation is always higher than the one leading to Hopf bifurcation. So, in this study, we focus on the values of the critical wave number k_{c2}, the critical Rayleigh number Ra_{c2} and the critical frequency ω_{c2} related to the Hopf bifurcation. The results of linear stability analysis for $Le = 30$ and $Le = 100$ and for $\psi = 0.1$ are presented in Tables 2 and 3. It can be observed in these two tables that the vibrations have a stabilizing effect and lead to an increase in the critical value of the thermal Rayleigh number. So the vibrations can be used to maintain the monocellular flow and then allow the separation of the binary mixture components over a wide range of thermal Rayleigh number. But it should be noted that the separation value decreases for the high values of Ra. It should be mentioned that vibrations reduce the critical wave number k_{c2} and the Hopf frequency ω_{c2}. This means that vibrations can also be used to decrease the number of convective cells at the transition from the monocellular flow to the multicellular flow.

Table 2
Effect of vibrations on the critical values of Rayleigh number Ra_{c2}, wave number k_{c2} and frequency ω_{c2} associated with transition from monocellular to multicellular flow for $Le = 30$, $\varepsilon = 0.5$, $B = 10^{-6}$ (order 4 Galerkin method)

Rv	Ra_{c2}	k_{c2}	ω_{c2}
0	31.48	2.84	2.21
10	36.51	2.63	2.25
20	40.96	2.45	2.25
30	44.98	2.30	2.24
40	48.67	2.18	2.23
50	52.08	2.08	2.21
60	55.28	2.00	2.20
70	58.28	1.92	2.18
80	61.12	1.86	2.17
90	63.83	1.81	2.16
100	66.41	1.76	2.15

Table 3
Effect of vibrations on the critical values of Rayleigh number Ra_{c2}, wave number k_{c2} and frequency ω_{c2} associated with transition from monocellular to multicellular flow for $Le = 100$, $\varepsilon = 0.5$, $B = 10^{-6}$ (order 4 Galerkin method)

Rv	Ra_{c2}	k_{c2}	ω_{c2}
0	27.67	3.04	1.44
10	32.29	2.81	1.43
20	36.44	2.61	1.40
30	40.21	2.44	1.37
40	43.67	2.3	1.34
50	46.88	2.18	1.32
60	49.88	2.08	1.30
70	52.70	2.00	1.28
80	55.37	1.92	1.26
90	57.91	1.86	1.25
100	60.33	1.81	1.24

7. 2D numerical simulations

The averaged equations (Eq. 6) with the associated boundary conditions were solved using the finite element method and the spectral collocation method. The influence of vibrations on the onset of convection was investigated for a cell of aspect ratio $A = 20$ for $Le = 30$ and $Le = 100$, $\varepsilon = 0.5$, (B is fixed to 10^{-6}).

7.1. Stability of the equilibrium solution

It was observed that the critical parameters of the bifurcations differed very little between the case $A = 20$ and the case of a cell of infinite horizontal extension. A structured mesh 150×30 was used for the finite element method for $A = 20$ and 120×20 collocation points for the spectral method.

For the onset of stationary convection, the results for $Le = 30$ and $Le = 100$ are presented below. For $Le = 30$ and $\psi = 0.03$, without vibrations ($Rv = 0$), the critical parameters $Ra_c = 12.92$, $k_c = 1.356$ are obtained from the linear stability analysis. For the same values of Le and ψ but with vibrations ($Rv = 10$), we obtain $Ra_c = 13.36$, $k_c = 0$ from the linear stability analysis, so the critical wave number is zero, which means that the flow at the onset of convection is monocellular. To confirm this result, we used the direct numerical simulation to study the case $Le = 30$ and $\psi = 0.03$ for a value of Ra close to the critical value ($Ra = 13.5$) first without vibration ($Rv = 0$) and then with vibrations ($Rv = 10$). Fig. 4(a) shows the streamlines and isoconcentrations for $Rv = 0$. In this case, the flow is multicellular and we cannot use the horizontal cell to separate the components of a binary mixture. Fig. 4(b) shows the streamlines and isoconcentrations for the same values of all parameters but with

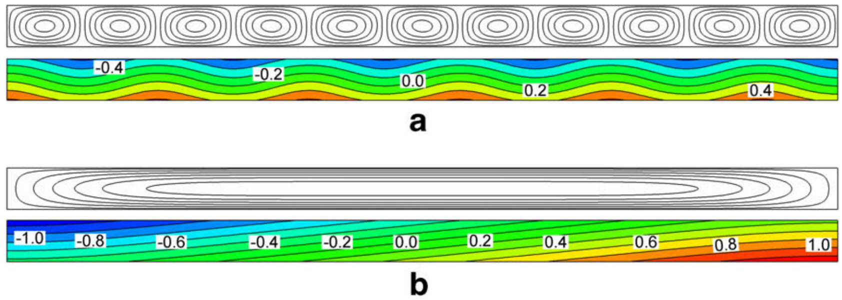

Fig. 4. Streamlines and isoconcentrations for $Le = 30$, $\psi = 0.03$, $Ra = 13.5$ (a) $Rv = 0$ (without vibration) (b). $Rv = 10$. A multicellular flow is obtained at the transition from the equilibrium solution for $Rv = 0$, whereas a monocellular flow is obtained for $Rv = 10$.

vibrations. It can be observed that vibrations modify the structure of the flow from multicellular to monocellular, leading to the stratification of the concentration field and the separation of the binary mixture components, but this separation is not very strong because the value of Ra is close the stability threshold and is far from $R_{opt} = 24/(Le\psi)$. Similar results were obtained for $Le = 100$ and $\psi = 0.01$ (Fig. 5).

7.2. Stability of the monocellular flow

In the results presented below, the values of B and ε were fixed at 10^{-6} and 0.5, respectively. As in the previous part, the results for $Le = 30$ and 100 are presented.

For $Le = 30$ we obtain a monocellular flow at the onset of convection for $\psi = \psi_{mono} = 0.044$ if $Rv = 0$ and for $\psi = \psi_{mono} = 0.014$ if $Rv = 50$. We present here some results for a value of ψ higher than these two values. We use $\psi = 0.1$. The stability analysis shows that, without vibrations, the monocellular flow loses its stability, via a Hopf bifurcation, for the critical parameters $Ra_{c2} = 31.48$ and $\omega_{c2} = 2.84$. For the same mixture ($\psi = 0.1$) under vibrations characterized by a modified vibrational Rayleigh number $Rv = 50$, the monocellular flow loses its stability for $Ra_{c2} = 52.08$ and $\omega_{c2} = 2.21$. These results were confirmed by the direct numerical simulations.

Fig. 6(a) shows the instantaneous streamline and isoconcentrations at a given time during the oscillation for $Ra = 31.4$. This value of Ra corresponds to the transition from monocellular flow to multicellular flow without vibrations ($Rv = 0$).

Fig. 6(b) shows the streamlines and the isoconcentrations for the same mixture with the same parameters but with vibrations

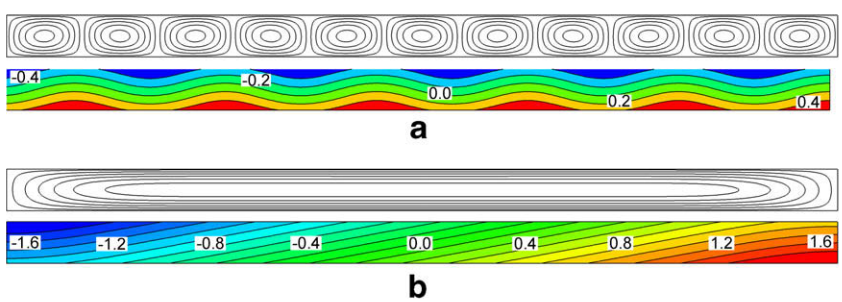

Fig. 5. Streamlines and isoconcentrations for $Le = 100$, $\psi = 0.01$, $Ra = 12.5$ (a) $Rv = 0$ (without vibration) (b). $Rv = 10$. A multicellular flow is obtained at the transition from the equilibrium solution for $Rv = 0$, whereas a monocellular flow is obtained for $Rv = 10$.

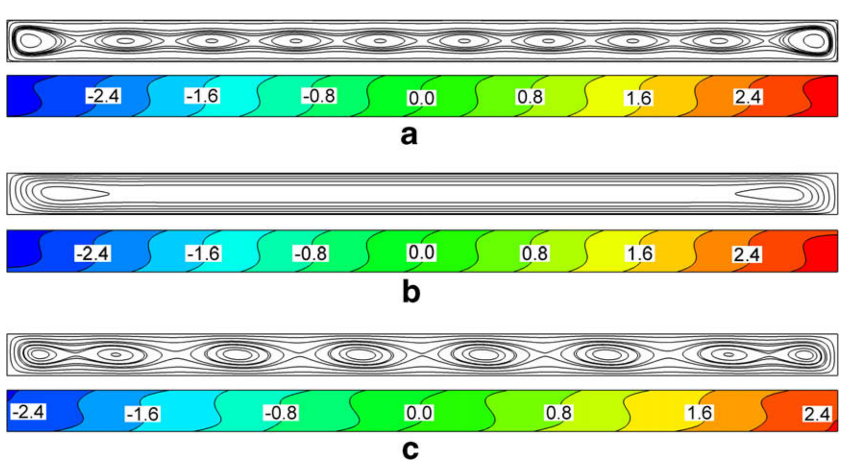

Fig. 6. $Le = 30$, $\psi = 0.1$: instantaneous streamline and isoconcentrations at a given time during the oscillation for (a) $Ra = 31.4$, and $Rv = 0$ (c) $Ra = 52.2$ and $Rv = 50$. (b) Streamline and isoconcentrations for $Ra = 31.4$, and $Rv = 50$. The vibrations maintain the monocellular flow for a higher value of the Rayleigh number ((a) and (b)).

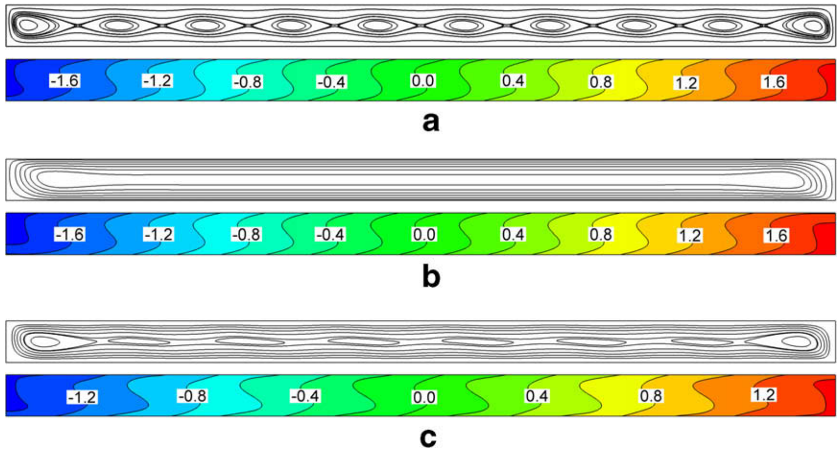

Fig. 7. $Le = 100$, $\psi = 0.1$: instantaneous streamline and isoconcentrations at a given time during the oscillation for (a) $Ra = 27.3$, and $Rv = 0$ (c) $Ra = 45.3$ and $Rv = 50$. (b) Streamline and isoconcentrations for $Ra = 27.3$, and $Rv = 50$.

($Rv = 50$). It was noted that, with vibrations, the monocellular flow could be maintained for a higher value of the Rayleigh number leading to the separation of the species between the left and the right vertical walls of the horizontal cell. But this separation is not very strong because the value of $RaLe\psi = 94.2$ is far from the optimal value: 24. This monocellular flow remains stable up to $Ra = 52.2$. Fig. 6(c) the instantaneous streamline and isoconcentrations at a given time during the oscillation at the transition from monocellular flow to multicellular flow ($Ra = 52.2$) for $Rv = 50$.

For $Le = 100$, we observe the same behavior as for the case $Le = 30$. As shown in Table 3, for $Le = 100$, the following critical parameters are obtained from the linear stability analysis: $Ra_{c2} = 27.67$ and $\omega_{c2} = 1.44$ for $Rv = 0$ and $Ra_{c2} = 46.88$ and $\omega_{c2} = 1.32$ for $Rv = 50$.

Fig. 7(a) shows the instantaneous streamline and isoconcentrations at a given time during the oscillation for the transition from monocellular flow to multicellular flow for $Rv = 0$. Under certain conditions, the vibrations lead to a change from multicellular to monocellular flow (Fig. 7(b)). This monocellular flow remains stable for a Rayleigh number much higher than the one obtained without vibrations. Thus the vibrations allow separation for high values of the Rayleigh number. When the Rayleigh number is increased slightly, the monocellular flow loses its stability to give a multicellular flow (Fig. 7(c)). The numerical results are then in good agreement with the analytical results.

The Hopf bifurcation frequency was found numerically for all the cases studied. For example for $Le = 30$, $\psi = 0.1$, $Ra = 52.2$, and $Rv = 50$, it was observed that the flow remains oscillatory for all the computing times considered. The value of the critical frequency obtained by the linear stability analysis is $\omega_{c2} = 2.21$. Using the Fourier transform of the horizontal component of the velocity at one point of the domain, we obtained a numerical critical frequency $\omega_{c2num} = 2.19$. For all the cases studied a good agreement was found between the theoretical and numerical results.

8. Conclusions

The Soret-driven convection in a large aspect ratio horizontal porous layer, heated from below, saturated by a binary fluid and subjected to vertical high-frequency vibrations was studied. The influence of vertical vibrations on the onset of convection and on the stability of the monocellular flow obtained for particular ranges of the physical parameters was investigated. We considered the case of high-frequency, small-amplitude vibrations so that a formulation using time averaged equations could be used. From the stability analysis of the rest solution obtained under the effect of vertical vibrations, it was observed that vertical high-frequency vibrations had a stabilizing effect on the convective flow. It was found that vibrations could be used to decrease the value of the separation ratio beyond which the flow at the onset of convection became monocellular, allowing separation of the components in the horizontal cell for a wide range of positive separation-ratio binary mixtures.

Analytical and numerical techniques were used to study the stability of the mono-cellular flow obtained, for $\psi \geqslant \psi_{mono} > 0$, when the equilibrium solution lost its stability. The direct nonlinear numerical simulations performed using the finite element method and the spectral collocation method corroborate the results of the linear stability analysis and allow the study of the flow structures which appear after the bifurcation. It was highlighted that the monocellular flow associated with a stratified concentration field led to a horizontal separation of the binary mixture components. It was observed that vibrations had a stabilizing effect leading to an increase in the critical value of the Rayleigh number corresponding to the transition between monocellular and multicellular flow. Thus vertical vibrations allow species separation over a wider range of Rayleigh numbers. It should also be noted that vibrations reduce the critical wave number k_{c2}, the Hopf frequency ω_{c2} and ψ_{mono} .

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